LAMINAR FLOW OF VAPOR BETWEEN PARALLEL DISKS WITH INTENSE UNIFORM ASYMMETRIC SUBLIMATION

P. A. Novikov, L. Ya. Lyubin, and V. I. Balakhonova

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The method of matching outer and inner asymptotic expansions is applied to the singular problem of laminar flow between sublimating disks at large injection Reynolds numbers.

Self-similar solutions of the problem of flows of incompressible fluid in a plane channel with porous walls and uniform symmetric injection or suction were considered in [1-3]. Self-similar solutions to the problem of flows between parallel disks sublimating at equal rate into the gap were obtained by Samsonov et al. [4], who also found the asymptotic form of these solutions for small injection Reynolds numbers R and predicted the limiting form of the solution for symmetric injection at $R=\infty$.

In the present paper we consider the asymptotic form of the solution of the problem of vapor flows between disks sublimating into the gap at equal rates j_* and j_{1*}, corresponding to large R and R_1.

We assume that the relative change in vapor density ρ_* at distances commensurable with the disk radius r_{0*} (which is the linear scale of flows in the plane of symmetry of the gap) is negligibly small and, hence, the approximation of an incompressible fluid is permissible.

The equations of motion and continuity reduced to dimensionless form and the boundary conditions have the form

$$v\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}=-\frac{\partial P}{\partial r}+\frac{1}{R}\left\{\frac{\partial^2 v}{\partial z^2}+\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rv)\right]\right\},$$
$$v\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{\partial P}{\partial z}+\frac{1}{R}\left[\frac{\partial^2 w}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)\right], \tag{1}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rv)+\frac{\partial w}{\partial z}=0, \tag{2}$$

$$w=1,\ v=0 \text{ when } z=-1;\ w=\chi,\ v=0 \text{ when } z=1. \tag{3}$$

As length and velocity scales we use half the distance between the disks h_* and the vapor velocity j_*/ρ_* on the surface of the lower disk ($z_*=-h_*$). The ratio of the sublimation rates is characterized by the parameter $\chi=j_{1*}/j_*$.

We can assume that the boundary conditions over the gap contour are uniform and the dimensionless disk radius $r_0\gg1$. In view of this, we considered that we could adopt the asymptotic approach corresponding to the treatment of a model problem for disks of infinite radius ($r_0\to\infty$), i.e., reject investigation of the flow details in the region of the boundary layer formed in the vicinity of the cylindrical surface $r=r_0$. We can then use the self-similar solution found in [4] for the considered problem

$$w=w(z);\ v=-\frac{1}{2}r\frac{dw}{dz}. \tag{4}$$

Function w(z) satisfied the equation

$$\frac{1}{R}\frac{d^4w}{dz^4}=w\frac{d^3w}{dz^3} \tag{5}$$

and the boundary conditions

$$w(0)=1,\ w'(0)=0,\ w(2)=\chi,\ w'(2)=0. \tag{6}$$

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk.
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At large R Eq. (5) is singularly perturbed. The solution of the shortened equation corresponding to it

$$ww''' = 0 \tag{7}$$

in correspondence with the terminology used in [5] is stable to the left on the left of the inflow plane [i.e., at $z < z_0$, $w(z^0) = 0$) and stable to the right on the right of this plane. Hence, the boundary layer, within which viscous dissipation effects appear and there is a sharp change in the function w'' characterizing the slope of the curve of the radial component of the velocity v, in the considered flow regimes is situated in the vicinity of the inflow plane: $z = z^0$. When $\chi = O(1)$ the thickness of this layer is of the order $\varepsilon = |R|^{-1/2}$. In view of this, we use the dilatation transformation $\zeta = (z - z^0)/\varepsilon$ at the point $z = z^0$, and the problem is solved by matching two outer and one inner asymptotic solutions [6].

The inner asymptotic expansion is constructed in the form

$$w(z,\ \varepsilon) = \varepsilon W_1(\zeta) + \varepsilon^2 W_2(\zeta) + \cdots. \tag{8}$$

The outer asymptotic expansions for the region $0 \le z < z^0$ and $z^0 < z \le 2$ are accordingly sought in the form

$$\begin{aligned} w(z,\ \varepsilon) &= w_{10}(z) + \varepsilon w_{11}(z) + \varepsilon^2 w_{12}(z) + \cdots, \\ w(z,\ \varepsilon) &= w_{20}(z) + \varepsilon w_{21}(z) + \varepsilon^2 w_{22}(z) + \cdots. \end{aligned} \tag{9}$$

The coordinate z^0 of the inflow plane is also sought in the form of an expansion of powers of the small parameter ε

$$z^0 = z_0^0 + \varepsilon z_1^0 + \varepsilon^2 z_2^0 + \cdots. \tag{10}$$

In the shortened Eq. (7) there is degeneracy due to annulment of the third derivative. Hence, the terms of the outer asymptotic expansions satisfying boundary conditions (6) can be written immediately in the following form:

$$\begin{aligned} &w_{10} = c_{10}\,\frac{(z+1)^2}{2} + 1;\ w_{1k} = c_{1k}\,\frac{(z+1)^2}{2}; \\ &w_{20} = c_{20}\,\frac{(z-1)^2}{2} - \chi;\ w_{2k} = c_{2k}\,\frac{(z-1)^2}{2}. \end{aligned} \tag{11}$$

To determine the unknown coefficients c_{jk} and the constants of integration of the equations corresponding to terms of the inner asymptotic expansion (8), we use the matching conditions in the vicinity of the inflow plane $z = z^0$:

on the left

$$\begin{aligned} &[\varepsilon W_1(\zeta) + \varepsilon^2 W_2(\zeta) + \varepsilon^3 W_3(\zeta) + \cdots]_{\zeta\to-\infty} \sim 1 + \\ &+ \left\{\frac{(1+z_0^0)^2}{2} + \varepsilon(1+z_0^0)(z_1^0+\zeta) + \varepsilon^2\left[(1+z_0^0)\,z_2^0 + \frac{(z_1^0)^2}{2} + z_1^0\zeta + \frac{\zeta^2}{2}\right] + \right. \\ &\left. + \varepsilon^3[(1+z_0^0)\,z_3^0 + z_1^0 z_2^0 + z_2^0\zeta] + \cdots\right\}(c_{10} + \varepsilon c_{11} + \cdots) \end{aligned} \tag{12}$$

and on the right

$$\begin{aligned} &[\varepsilon W_1(\zeta) + \varepsilon^2 W_2(\zeta) + \varepsilon^3 W_3(\zeta) + \cdots]_{\zeta\to\infty} \sim -\chi + \\ &+ \left\{\frac{(1-z_0^0)^2}{2} - \varepsilon(1-z_0^0)(z_1^0+\zeta) - \varepsilon^2\left[(1-z_0^0)\,z_2^0 - \frac{(z_1^0)^2}{2} - z_1^0\zeta - \frac{\zeta^2}{2}\right] - \right. \\ &\left. - \varepsilon^3[(1-z_0^0)\,z_3^0 - z_1^0 z_2^0 - z_2^0\zeta] - \cdots\right\}(c_{20} + \varepsilon c_{21} + \cdots). \end{aligned} \tag{13}$$

From the Taylor expansion of the function $w(z,\ \varepsilon)$ in the vicinity of $z = z^0$

$$w(z,\ \varepsilon) = w'(z^0,\ \varepsilon)(z - z_0) + w''(z^0,\ \varepsilon)\,\frac{(z-z^0)^2}{2} + \cdots$$

and the asymptotic expansion

$$w'(z^0,\ \varepsilon) = -b_0 - \varepsilon b_1 - \varepsilon^2 b_2 - \cdots$$

it follows that

$$W_1 = -b_0\zeta. \tag{14}$$

Thus, substitution of (8) in (5) gives

$$\frac{d^4 W_2}{d\zeta^4} + b_0\zeta\,\frac{d^3 W_2}{d\zeta^3} = 0.$$

The solution of the last equation satisfying the condition $W_2(0) = 0$ has the form

$$W_2 = A_2 \frac{\zeta^2}{2} + B_2 \sqrt{\frac{\pi}{2b_0}} \left\{\left(\frac{\zeta^2}{2} + \frac{1}{\sqrt{\pi b_0}}\right) \Phi\left(\sqrt{\frac{b_0}{2}}\,\zeta\right) + \frac{\zeta}{\sqrt{2\pi b_0}} \exp\left(-\frac{b_0}{2}\zeta^2\right) - \sqrt{\frac{2}{\pi b_0}}\,\zeta\right\} - b_1\zeta. \tag{15}$$

Here

$$\Phi(x) = \int_0^x e^{-x_1^2}\,dx_1.$$

By substituting expressions (14) and (15) in the matching conditions (12) and (13) and equating coefficients of equal powers of ε we find the coefficients

$$c_{10} = -\frac{\varkappa^2}{2};\; c_{20} = \frac{\varkappa^2}{2\chi};\; c_{11} = 0;\; c_{21} = 0;\; c_{12} = -\frac{\varkappa^3(3-\chi)}{8\chi\sqrt{\pi}};$$

$$c_{22} = \frac{\varkappa^3(3\chi-1)}{8\chi^3\sqrt{\pi}};\; A_2 = \frac{\varkappa^2(1-\chi)}{4\chi};$$

$$B_2 = -\varkappa b_1;\; z_0^0 = \frac{1-\chi}{\varkappa};\; z_1^0 = 0;\; z_2^0 = -\frac{1-\chi}{2\chi\sqrt{\pi}};$$

$$b_0 = \varkappa;\; b_1 = -\frac{\varkappa^2}{4\chi}\sqrt{\frac{2\varkappa}{\pi}};\; \varkappa = \chi + 1.$$

The leading terms of the inner asymptotic expansion (8) satisfy equations of the form ($k \geq 3$)

$$\frac{d^4W_k}{d\zeta^4} + \varkappa\zeta\,\frac{d^3W_k}{d\zeta^3} = \sum_{j=2}^{k-1} W_j\,\frac{d^3W_{k-j}}{d\zeta^3}.$$

The solutions of these equations could be found successively in the form of quadratures. With increase in k, however, the right-hand side of the equation becomes a very unwieldy combination of exponential functions and functions of the type $\Phi^n(\beta\zeta)$. Hence, we determine only W_3; i.e., we seek the solution, satisfying the condition $W_3(0) = 0$, of the equation

$$\frac{d^4W_3}{d\zeta^4} + \varkappa\zeta\,\frac{d^3W_3}{d\zeta^3} = \frac{\varkappa^5}{16\chi^2}\sqrt{\frac{2\varkappa}{\pi}}\exp\left(-\frac{\varkappa}{2}\zeta^2\right) \times$$

$$\times\left\{\sqrt{\frac{2\varkappa}{\pi}}\,\zeta + \frac{1-\chi}{2}\zeta^2 + \varkappa\left[\left(\frac{\zeta^2}{2} + \frac{1}{\sqrt{\pi\varkappa}}\right)\Phi\left(\sqrt{\frac{\varkappa}{2}}\,\zeta\right) + \frac{\zeta}{\sqrt{2\pi\varkappa}}\exp\left(-\frac{\varkappa}{2}\zeta^2\right) - \sqrt{\frac{2}{\pi\varkappa}}\cdot\zeta\right]\right\}.$$

The corresponding solution has the form

$$W_3 = \frac{\varkappa^5}{16\chi^2}\left\{\frac{1}{2}\,\frac{1-\chi}{\varkappa^2}\left[\frac{4}{3}\,\frac{1}{\sqrt{\pi}}\sqrt{\frac{2}{\varkappa}}\left(1 - \exp\left(-\frac{\varkappa}{2}\zeta^2\right)\right) - \right.\right.$$

$$\left. - \zeta\Phi\left(\sqrt{\frac{\varkappa}{2}}\,\zeta\right)\right] - \left(\frac{1}{2\pi} + \frac{7}{12}\,\frac{1}{\sqrt{\pi}}\right)\frac{\Phi(\sqrt{\varkappa}\zeta)}{\varkappa\sqrt{\varkappa}} +$$

$$+\left(\frac{2}{\pi} + \frac{1}{3\sqrt{\pi}}\right)\left[\frac{\zeta^2\Phi(\sqrt{\varkappa}\zeta)}{2\sqrt{\varkappa}} + \frac{\zeta\exp(-\varkappa\zeta^2)}{2\sqrt{\pi\varkappa}}\right] -$$

$$-\frac{1}{6}\sqrt{\frac{2}{\pi\varkappa}}\,\frac{1}{\varkappa}\Phi\left(\sqrt{\frac{\varkappa}{2}}\,\zeta\right) - \frac{1}{2\varkappa}\left(\frac{1}{2} + \frac{1}{\sqrt{\pi}}\right)\times$$

$$\left.\times\left[\zeta\Phi^2\left(\sqrt{\frac{\varkappa}{2}}\,\zeta\right) + 2\,\frac{\sqrt{2}}{\sqrt{\pi\varkappa}}\Phi\left(\sqrt{\frac{\varkappa}{2}}\,\zeta\right)\exp\left(-\frac{\varkappa}{2}\zeta^2\right)\right]\right\} +$$

$$+ A_3\frac{\zeta^2}{2} + \frac{B_3}{2}\sqrt{\frac{\pi}{2\varkappa}}\left[\left(\zeta^2 + \frac{1}{\varkappa}\right)\Phi\left(\sqrt{\frac{\varkappa}{2}}\,\zeta\right)\sqrt{\frac{2}{\pi\varkappa}}\,\zeta\exp\left(-\frac{\varkappa}{2}\zeta^2\right)\right] - b_2\zeta. \tag{16}$$

From the matching conditions (12) and (13) we find the coefficients

$$A_3 = 0;\; B_3 = -\frac{\varkappa^5}{16\chi}\sqrt{\frac{2}{\pi}}\left(\frac{2}{\pi} + \frac{1}{3\sqrt{\pi}}\right);\; b_2 = \frac{\varkappa^2}{2\chi\sqrt{\pi}} -$$

$$-\frac{\varkappa^3}{32\chi^2}\left[1 - \chi + \left(\frac{1}{2} + \frac{1}{\sqrt{\pi}}\right)\varkappa\right];\; c_{13} = \frac{\varkappa^6}{32\chi^2\sqrt{\varkappa}}\times$$

$$\times\left[\frac{2}{3}\sqrt{\frac{2}{\pi}}\frac{(1-\chi)^2}{\varkappa^2}+\frac{1}{6}\sqrt{\frac{2}{\pi}}+\frac{3}{2}\frac{1}{\pi}+\frac{3}{4}\frac{1}{\sqrt{\pi}}\right];$$

$$c_{23}=-\frac{c_{13}}{\chi^2};\quad z_3^0=-\frac{1}{6\chi}\sqrt{\frac{2}{\pi}}\frac{1-\chi}{\sqrt{\varkappa}}.$$

Using the inner asymptotic expansion (8), we can, by determining the first three derivatives of the function w at the point $z=z^0$, reduce the considered two-point boundary-value problem to two Cauchy problems for Eq. (5) and the equation

$$\frac{1}{R}\frac{d^4u}{dy^4}=-u\frac{d^3u}{dy^3},\tag{17}$$

obtained from (5) by the transformation $y=-z$, $u=w$.

The initial conditions of the first problem are

$$\begin{aligned}w(z^0)=0;\ w'(z^0)=-\varkappa+\frac{1}{\sqrt{R}}\frac{\varkappa^2}{4\chi}\sqrt{\frac{2\varkappa}{\pi}}+\\ +\frac{1}{R}\left\{\frac{\varkappa^4}{16\pi\chi^2}(\sqrt{2}-1)\left(\frac{\sqrt{2}}{3}-\frac{2}{\sqrt{\pi}}\right)+\right.\\ \left.+\frac{\varkappa^3}{32\chi^2}\left[1-\chi+\left(\frac{1}{2}+\frac{1}{\sqrt{\pi}}\right)\varkappa\right]-\frac{\varkappa^2}{2\chi\sqrt{\pi}}\right\}+O(R^{-3/2});\\ w''(z^0)=(1-\chi)\left[\frac{\varkappa^2}{4\chi}-\frac{1}{\sqrt{R}}\frac{\varkappa^3}{48\chi^2}\sqrt{\frac{2\varkappa}{\pi}}+O\left(\frac{1}{R}\right)\right];\\ w'''(z^0)=\sqrt{R}\frac{\varkappa^3}{4\chi}\sqrt{\frac{2\varkappa}{\pi}}-\frac{\varkappa^5}{16\pi\chi^2}\times\left[\frac{2}{\sqrt{\pi}}(\sqrt{2}-1)+\frac{\sqrt{2}+1}{3}\right]+O(R^{-1/2});\\ z^0=\frac{1-\chi}{1+\chi}-\frac{1}{R}\frac{1-\chi}{2\sqrt{\pi}\chi}-\frac{1}{R^{3/2}}\frac{1-\chi}{8}+O(R^{-2}).\end{aligned}\tag{18}$$

The initial conditions of the second problem are

$$u(-z^0)=0;\ u'(-z^0)=w'(z^0);\ u''(-z^0)=-w''(z^0);\ u'''(-z^0)=w'''(z^0).\tag{19}$$

Problems (5), (18) and (17), (19) are solved in the range from z^0 to 1 and from $-z^0$ to 1, respectively.

The least accurately prescribed initial conditions are $w''(z^0)$, $w'''(z^0)$, and $u''(-z^0)$, $u'''(-z^0)$. Hence, for more accurate calculations at not very large $|R|$ these conditions should be made more accurate, which can be done without determining the next term of the inner asymptotic expansion (W_4).

If, for the functions w'' and u'' outside the boundary layer formed near the inflow plane $z=z^0$, we introduce the symbols w''_∞ and u''_∞, it is obvious that when accurate initial conditions are used we would have $w''_\infty=c_2$ and $u''_\infty=-c_1$, where

$$c_j=c_{j0}+\frac{1}{R}c_{j2}+\frac{1}{R^{3/2}}c_{j3}+O(R^{-2})\quad(j=1,\ 2).\tag{20}$$

Thus, if we select for correction of the initial conditions a sufficiently large value of $R=R_0$, we can assume that the corresponding correction to $w''(z^0)$ is

$$\Delta w''(z^0,\ R_0)\approx\frac{c_2-c_1-w''_\infty(R_0)-u''_\infty(R_0)}{2}.$$

To calculate the right-hand side we use the results of a computer solution of Cauchy problems with initial conditions determined accurately to the terms written in (18) and (19). Coefficients c_1 and c_2 are found accurate to the terms written in (20). The dependence of the corresponding correction on R is given by the asymptotic equation

$$\Delta w''(z^0,\ R)=\frac{W''_4(0)}{R}+O(R^{-3/2}).\tag{21}$$

Thus, after determination of this correction for some value of $R=R_0$ we can find $W''_4(0)$ with corresponding accuracy

$$W''_4(0)=R_0\Delta w''(z^0,\ R_0)+O(R^{-1/2})$$

and use formula (21) to calculate the correction for different R and a fixed value of the parameter χ.

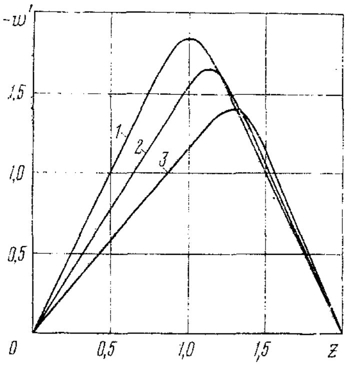

Fig. 1. Profiles of functions $-w'(z)$.

To make $w'''(z^0)$ more accurate we can use a linear interpolation. For this purpose a computer is used to solve two Cauchy problems corresponding to Eq. (5) with initial conditions (18) and the conditions obtained from (18) by the introduction of a small perturbation δ of $w'''(z^0)$, i.e., replacement of this quantity by the number $w'''(z^0)+\delta$. The values of the second derivative obtained in this case outside the boundary layer $w''_\infty[w'''(z^0)]$ and $w''_\infty[w'''(z^0)+\delta]$ are used to calculate the correction to $w'''(z^0)$

$$\Delta w'''(z^0) = \Delta u'''(-z^0) = \frac{\{c_2 - w''_\infty[w'''(z^0)]\}\,\delta}{w''_\infty[w'''(z^0)+\delta] - w''_\infty[w'''(z^0)]}.$$

The corrections for other values of R (with χ fixed) can be obtained from the equation

$$\Delta w'''(z^0, R) = \frac{W_4'''(0)}{\sqrt{R}} + O(R^{-1}), \tag{22}$$

where $W_4'''(0) = \sqrt{R_0}\Delta w'''(z^0, R_0) + O(R_0^{-1/2})$.

The profiles of function $w'(z)$ (Fig. 1), which is proportional to the radial component of the velocity v, were plotted from the results of solution of the corresponding Cauchy problems with the initial conditions corrected by the asymptotic equations (21) and (22). Curves 1, 2, and 3 correspond to the regimes $R=-50$, $\chi=1$; $R=-100$, $\chi=3/4$; $R=-50$, $\chi=1/2$. The calculated values of $w(-1)$ and $w(1)$ differ from -1 and χ, respectively, by $(1-2)\cdot 10^{-3}$. The deviation of the calculated values of $w'(\pm 1)$ from 0 is of the same order.

It was assumed above that the parameter χ characterizing the asymmetry of sublimation is on the order of 1. It is apparent from the structure of the outer and inner expansions that the plotted asymptotic curve has a meaning if the conditions $|R|^{-1/2} \ll \chi \ll |R|^{1/2}$ are satisfied.

At the same time, for several areas of modern technology (sublimation vacuum drying) a study of similar processes involving unilateral sublimation, when $\chi=0$, is important. The construction of the asymptotic forms corresponding to large $|R|$ in this case is, on one hand, a much more complex problem, than when $\chi\sim 1$ and, on the other hand (there is no special need for this), when $\chi=0$ the analysis of all the regimes reduces to construction of a single-parameter family of solutions of Eq. (5) satisfying the conditions

$$w(0)=1;\; w'(0)=0;\; w(2)=0;\; w'(2)=0.$$

The variable parameter is the number R contained in Eq. (5). This boundary-value problem is easily reduced to a Cauchy problem for an equation not containing the parameter

$$\frac{d^4f}{dz^4} = f\,\frac{d^3f}{dz^3}, \tag{23}$$

which was obtained by the introduction of new variables (similar to those proposed in [2]) $z=\beta(2-z)$, $w(z) = w\left(2-z/\beta\right) = -\alpha f(z)$, $\beta=\alpha R$. The initial conditions containing the variable parameter γ have the form

$$f(0)=0,\; f'(0)=0,\; f''(0)=1,\; f'''(0)=-\gamma, \tag{24}$$

i.e., $f=f(z,\gamma)$, where points on the positive semi-axis $0<\gamma<\infty$ correspond to the parameter γ characterizing the specific solution. For each selected value of γ, Eq. (23) is integrated in the limits from 0 to z^0, where z^0 is selected from the condition $f'(z^0)=0$. It is obvious that $z^0=2\beta$ and $R=-\beta f(2\beta)$ will correspond to the obtained solution.

Thus, taking several values of parameter γ we can obtain a fairly complete representation of flows of vapor with unilateral sublimation characterized by R in the range $-\infty<R<0$.

NOTATION

r_*, z_*, cylindrical coordinates; v_*, w_*, radial and transverse velocity components; $2h_*$, distance between disks; j_*, j_{1*}, rates of sublimation of lower ($z=-1$) and upper ($z=1$) disks; P_*, pressure; ρ_*, density; μ_*, dynamic viscosity; $w_{ik}(i=1, 2)$, terms of outer asymptotic expansions (9); $c_{ik}(i=1, 2)$, coefficients of (11); W_k, terms of inner asymptotic expansion (8); z^0, dimensionless coordinate of inflow plane; z_k^0, terms of expansion (10); $R=-j_*h_*/\mu_*$; $R_1=-j_{1*}h_*/\mu_*$; $\chi=j_{1*}/j_*$; $v=v_*\rho_*/j_*$; $w=w_*\rho_*/j_*$; $r=r_*/h_*$; $z=z_*/h_*$; $P=P_*\rho_*/j_*^2$.

LITERATURE CITED

1. A. S. Berman, "Laminar flow in channels with porous walls," J. Appl. Phys., 24, No. 9, 1232-1235 (1953).
2. R. M. Terrill, "Laminar flow in a uniformly porous channel," Aeronaut. Q., 15, 299-310 (1964).
3. R. M. Terrill, "Laminar flow in a uniformly porous channel with large injection," Aeronaut. Q., 16, 323-332 (1965).
4. V. V. Samsonov and D. A. Labuntsov, "Analytical and experimental investigation of hydrodynamics of a vapor flow in a sublimation heat exchanger," Tr. Mosk. Énerg. Inst. Teplo. Massoob. Prots. Appar., No. 198 (1974).
5. A. B. Vasil'eva and V. F. Butuzov, Asymptotic Expansions of Solutions of Singularly Perturbed Equations [in Russian], Nauka, Moscow (1973).
6. M. Van Dyke, Perturbation Methods in Fluid Mechanics, Academic Press, New York (1964).

RHEOLOGICAL PROPERTIES OF HOMOGENEOUS FINELY DISPERSED SUSPENSIONS. STEADY FLOWS

Yu. A. Buevich and I. N. Shchelchkova

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Expressions are obtained for the rheological parameters (effective viscosity, force of interphase interaction, etc.) of a moderately concentrated suspension of spherical particles. The equations of motion of the suspension and of its phases are written.

A situation when the characteristic spatial scale of the average motion of a dispersed medium is far greater than its internal structural scale, so that it is natural to use the methods of the mechanics of continuous media to describe such motion, is very common in applications. Two fundamental problems arise in this case: to obtain the conservation equations describing the average flow of the phases of the medium and to formulate the rheological equations closing them. In connection with the wide prevalence of dispersed media in various fields of practical activity, both these problems have been discussed in a very large number of reports using the most varied theoretical and experimental methods for media of the most varied types.

For systems consisting of a continuous phase and discrete elements of a dispersed phase distributed in it, the first problem was formally solved in [1, 2] using the well-developed method of averaging of the local conservation equations, which are valid within the materials of the phases, over the ensemble of possible configurations of particles of the dispersed phase. (Bibliographies of research in this field are also presented in the cited reports.) The basic method of solving the second main problem was also indicated in [1, 2], but it was studied concretely only for steady streams of a monodispersed medium containing fine spherical particles in the case when their volume concentration is not too high, so that in averaging over the ensemble one can neglect the nonoverlapping of the solid spheres in the first approximation. In this case the suspension was analyzed as a macroscopic homogeneous "one-velocity" medium.* Analogous problems for the process of heat or mass exchange in a granular medium were considered in [3].

*The results of [1, 2] are presented in Russian in the following preprints: Yu. A. Buevich and I. N. Shchelchkova, "Continuous mechanics of monodispersed suspensions. Conservation equations," and Yu. A. Buevich, B. S. Endler, and I. N. Shchelchkova, "Continuous mechanics of monodispersed suspensions. Rheological equations of state," Preprint Nos. 72 and 85, Inst. Prikl. Mekh. Akad. Nauk SSSR, Moscow (1976) and (1977).

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